Class: XII

SESSION : 2022-2023

SUBJECT: Mathematics

SAMPLE QUESTION PAPER - 3

with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. The perpendicular distance of the point whose position vector is (1, 3, 5) from the line $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ is equal to: **[1]**

a) 8 b) 2

c) 1 d) 3

2. If θ is an acute angle and the vector $(\sin\theta)\hat{i} + (\cos\theta)\hat{j}$ is perpendicular to the vector $\hat{i} - \sqrt{3}\hat{j}$ then θ = **[1]**

a) $\frac{\pi}{4}$ b) $\frac{\pi}{5}$

c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

3. $\int \sec^5 x \tan x \, dx$ =? **[1]**

a) None of these b) $\frac{1}{5}\sec^5 x + C$

c) $5 \tan^5 x + C$ d) 5 log |cos x| + C

4. Area enclosed between by the curve $y^2 (2a - x) = x^3$ and the line x = 2a above x-axis is **[1]**

a) $\frac{3}{2}\pi a^2$ b) πa^2

c) $3\pi a^2$ d) $2\pi a^2$

5. The area of the region bounded by y = | x - 1| and y = 1 is **[1]**

a) 2 b) $\frac{1}{2}$

c) none of these d) 1

6. Let A and B be two events such that P(A) = $\frac{3}{8}$, P(B) = $\frac{5}{8}$ and P(A $\cup$ B) = $\frac{3}{4}$. Then $P(A|B) \cdot P(A'/B)$ is equal to [1]

a) $\frac{2}{5}$

b) $\frac{6}{25}$

c) $\frac{3}{10}$

d) $\frac{3}{8}$

7. If E_1 and E_2 are two independent events, then P $(E_1 \cap E_2)$ is equal to [1]

a) $P(E_1) + P(E_2)$

b) $P(E_1) + P(E_2) + P(E_1 + \cup E_2)$

c) $P(E_1)P(E_2)$

d) none of these

8. Objective function of an LPP is [1]

a) a function to be optimized

b) None of these

c) a constraint

d) a relation between the variables

9. The solution of the differential equation $\frac{dy}{dx}$ = 1 + x + y + xy ,y (0) = 0 is [1]

a) $y^2 = \exp\left(x + \frac{x^2}{2}\right) - 1$

b) $y^2 = 1 + \cos p\left(x + \frac{x^2}{2}\right)$

c) $y = \tan\left(x + \frac{x^2}{2}\right)$

d) $y = \tan(C + x + x^2)$

10. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a} + \vec{b}$ is collinear with $\vec{c}, \vec{b} + \vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + \vec{b} + \vec{c} =$ [1]

a) $\vec{a}$

b) $\vec{c}$

c) $\vec{b}$

d) None of these

11. $\int \frac{(1+\tan x)}{(1-\tan x)} dx = ?$ [1]

a) log |cos x – sin x| + C

b) log |cos x + sin x | + C

c) none of these

d) - log |cos x – sin x| + C

12. Solution of the differential equation $\frac{dy}{dx} + \frac{y}{x}$ = sin x is: [1]

a) x (y + cos x) = cos x + c

b) x (y + cos x) = sin x + c

c) x (y - cos x) = sin x + c

d) xy cos x = sin x + c

13. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is [1]

a) $y = 2x^2 - 4$

b) y = 2x

c) y = 2

d) y = 2x - 4

14. If $y = e^{1/x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{-e^{1/x}}{x^2}$

b) $e^{1/x} \log x$

c) $\frac{1}{x} \cdot e^{(1/x-1)}$

d) None of these

15. Consider the following statements **[1]**

i. The general solution of $\frac{dy}{dx}$ = f(x) + x is of the dx form y = g(x) + C, where C is an arbitrary constant.

ii. The degree of $\left(\frac{dy}{dx}\right)^2$ = f(x) is 2.

Which of the above statement(s) is/are correct?

a) Only (ii)

b) Only (i)

c) Both (i) and (ii)

d) Neither (i) nor (ii)

16. For any 2-rowed square matrix A, if A · (adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$ then the value of |A| is **[1]**

a) 8

b) 4

c) 0

d) 64

17. The principal value of $\tan^{-1}(-\sqrt{3})$ is **[1]**

a) $\frac{4\pi}{3}$

b) None of these

c) $\frac{2\pi}{3}$

d) $\frac{-\pi}{3}$

18. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is **[1]**

a) perpendicular to z-axis

b) parallel to z-axis

c) parallel to y-axis

d) parallel to x-axis

19. **Assertion (A):** A function f(x) = $\begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is discontinuous at x = 0. **[1]**

Reason (R): The function f(x) = $\begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$ is continuous for all values of x.

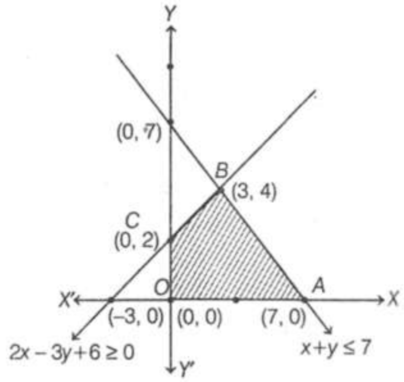
a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Objective function Z = 13x - 15y is minimised, subject to the constraints x + y ≤ 7, 2x - 3y + 6 ≥ 0, x ≥ 0, y ≥ 0. **[1]**



The minimum value of Z is - 21.
Reason (R): Optimal value of an objective function is obtained by comparing value of objective function at all comer points.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. If x = a sin t - b cos t, y = a cos t + b sin t, prove that $\frac{d^2y}{dx^2} = -\frac{x^2+y^2}{y^3}$ **[2]**

22. Find the angle b/w the line $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ **[2]**

OR

What is the angle between the vector $\vec{r} = (4\hat{i} + 8\hat{j} + \hat{k})$ and the x-axis?

23. Find the general solution for differential equation: $y\left(1-x^2\right)\frac{dy}{dx} = x\left(1+y^2\right)$ **[2]**

24. Write the interval for the principal value of function and draw its graph: $\operatorname{cosec}^{-1} x$. **[2]**

25. A class consists of 80 students; 25 of them are girls and 55 boys; 10 of them are rich and the remaining poor; 20 of them are fair complexioned. What is the probability of selecting a fair-complexioned rich girl? **[2]**

Section C

26. Find the area of the region bounded by y = $|x-1|$ and y = 1. **[3]**

OR

For any real t ,$x = \frac{e^t+e^{-t}}{2}, y = \frac{e^t-e^{-t}}{2}$ is a point on hyperbola $x^2 - y^2 = 1$. Find the area bounded by this hyperbola and the lines joining it's centre to the points corresponding to t_1 and $-t_1$.

27. Solved the linear programming problem graphically: **[3]**
Maximize Z = 60x + 15y
Subject to constraints
$x + y \leq 50$
$3x + y \leq 90$
$x, y \geq 0$

28. Show that the lines $\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$ and $\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$ do not intersect **[3]**

OR

Find the equation of the perpendicular drawn from the point P (2, 4, -1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Also, write down the coordinates of the foot of the perpendicular from P.

29. Sketch the graph $y = |x+1|$. Evaluate $\int_{-4}^{2} |x+1| dx$ What does the value of this integral represent on the graph? **[3]**

30. Evaluate: $\int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$ **[3]**

OR

Evaluate: $\int \frac{x^3 \sin^{-1} x^2}{\sqrt{1-x^4}} dx$

31. If x = ae^{t} (sin t + cos t) and y = ae^{t} (sin t – cos t), prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$. **[3]**

Section D

32. Find the inverse of the matrix (if it exists) given $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$ **[5]**

OR

For what values of a and b, the system of equations
2x + ay + 6z = 8
x + 2y + bz = 5
x + y + 3z = 4
has:

i. a unique solution
ii. infinitely many solutions
iii. no solution

33. Given $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$, $\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}), \hat{i}, \hat{j}, \hat{k}$ being a right handed orthogonal system of unit vectors in space, show that $\vec{a}, \vec{b}, \vec{c}$ is also another system. **[5]**

34. Evaluate the integral $\int_{1}^{2} \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$ using substitution. **[5]**

35. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

OR

Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by (x, y) ∈ R ⇔ x - y is divisible by n, is an equivalence relation on Z.

Section E

36. **Read the text carefully and answer the questions:** [4]

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Find the volume of the open box formed by folding up the cutting each corner with x cm.

(ii) Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(iii) Verify that volume of the box is maximum at x = 3 cm by second derivative test?

OR

Find the maximum volume of the box.

37. **Read the text carefully and answer the questions:** [4]

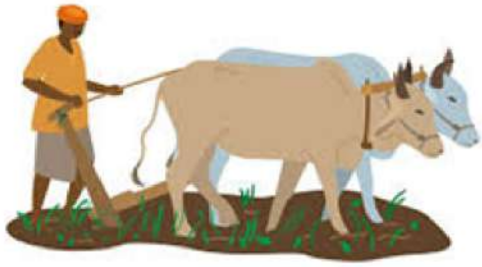
There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

38. **Read the text carefully and answer the questions:** [4]

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

October sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

(i) Find the combined sales of Masoor in September and October, for farmer Girish.

(ii) Find the combined sales of Urad in September and October, for farmer Ankit.

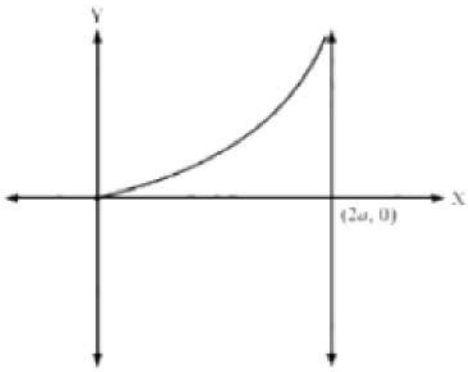
(iii) Find a decrease in sales from September to October.

OR

If both the farmers receive 2% profit on gross sales, then compute the profit for each farmer and for each variety sold in October.

SOLUTIONS

Section A

1. **(c)** 1
   Explanation: 1
2. **(c)** $\frac{\pi}{3}$
   Explanation: Since, the given two vectors are given as perpendicular then their dot product must be zero
   $((\sin\theta)\hat{i} + (\cos\theta)\hat{j}).(\hat{i} - \sqrt{3}\hat{j}) = 0$
   $\sin\theta - \sqrt{3}\cos\theta = 0$
   $\tan\theta = \sqrt{3}$
   Since θ is acute then, $\theta = \frac{\pi}{3}$
3. **(b)** $\frac{1}{5}\sec^5 x + C$
   Explanation: Given $= \int \sec^5 x \tan x dx = ?$
   So, $\int \sec^5 x \tan x dx = \int \sec^4 x(\sec x \tan x)dx$
   Let, $\sec x = z$
   $\Rightarrow \sec x \tan x dx = dz$
   $\int \sec^4 x(\sec x \tan x)dx$
   $= \int z^4 dz$
   $= \frac{z^5}{5} + c$
   $= \frac{\sec^5 x}{5} + c$
   where c is the integrating constant.
4. **(a)** $\frac{3}{2}\pi a^2$
   Explanation:
   

   $y^2 (2a - x) = x^3$

$$y=\sqrt{\frac{x^3}{2a-x}}$$

Let x = $2a\sin^2\theta$
dx = 4asinθcosθdθ

$$\text{Area} = \int_0^{2a}\sqrt{\frac{x^3}{2a-x}}dx$$

$$=\int_0^{\frac{\pi}{2}}\sqrt{\frac{\left(8a^3\right)\sin^6\theta}{(2a)\cos^2\theta}}\cdot(4a)\sin\theta\cos\theta d\theta$$

$$=8a^2\int_0^{\frac{x}{2}}\sqrt{\sin^6\theta}\sin\theta d\theta$$

$$=8a^2\left[\int_0^{\frac{\pi}{2}}\sin^4\theta d\theta\right]$$

$$=8a^2\left[\int_0^{\frac{\pi}{2}}\sin^2\theta\left(1-\cos^2\theta\right)d\theta\right]$$

$$=8a^2\left[\int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 2\theta}{2}\right)d\theta-\frac{1}{4}\int_0^{\frac{\pi}{2}}\sin^2 2\theta d\theta\right]$$

$$=8a^2\left[\frac{1}{2}[\theta]_0^{\frac{x}{2}}-\left[\frac{\sin 2\theta}{4}\right]_0^{\frac{x}{2}}\right]-\frac{1}{4}\left[\int_0^{\frac{\pi}{2}}\frac{1-\cos 4\theta}{2}d\theta\right]$$

$$= 8a^2\left[\left(\frac{\pi}{4}\right) - 0\right] - \frac{1}{4}\left[\frac{\pi}{4} - 0\right]$$

$$= 8a^2\left[\frac{\pi}{4} - \frac{\pi}{16}\right] = \frac{3}{2}\pi a^2$$

5. **(d)** 1

Explanation: Required area : $\left|\int_0^1 [(x-1)-(1-x)]dx\right| = 1$

6. **(b)** $\frac{6}{25}$

Explanation: Here, P(A) = $\frac{3}{8}$, P(B) = $\frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$

$\because$ P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$\Rightarrow P(A \cap B) = \frac{3}{8} + \frac{5}{8} - \frac{3}{4} = \frac{3+5-6}{8} = \frac{2}{8} = \frac{1}{4}$

$\because P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{5/8} = \frac{8}{20} = \frac{2}{5}$

And $P\left(A'/B\right) = \frac{P\left(A' \cap B\right)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{\frac{5}{8} - \frac{1}{4}}{\frac{5}{8}} = \frac{\frac{5-2}{8}}{\frac{5}{8}} = \frac{3}{5}$

$\therefore P(\frac{A}{B}) \cdot P\left(\frac{A'}{B}\right) = \frac{2}{5} \cdot \frac{3}{5} = \frac{6}{25}$

7. **(c)** $P\left(E_1\right)P(E_2)$

Explanation: We have, $P(E_1 \cap E_2) = P(E_1). P(\frac{E_2}{E_1})$

Since E_1 and E_2 are independents, therefore

$P = \left(\frac{E_2}{P(E_1) = P(E_2)}\right)$

$\therefore P(E_1 \cap E_2) = P(E_1). P(E_2)$

8. **(a)** a function to be optimized

Explanation: a function to be optimized

The objective function of a linear programming problem is either to be maximized or minimized i.e. objective function is to be optimized.

9. **(c)** $y = \tan\left(x + \frac{x^2}{2}\right)$

Explanation: We have ,

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\Rightarrow \frac{dy}{dx} = (1 + x)\left(1 + y^2\right)$$

$$\Rightarrow \frac{dy}{1+y^2} = (1 + x)dx$$

$$\Rightarrow \int \frac{dy}{1+y^2} = \int (1 + x)dx$$

$$\Rightarrow \int \frac{dy}{1+y^2} = \int (1 + x)dx$$

$$\Rightarrow \tan^{-1}y = x + \frac{x^2}{2} + c$$

Given $y(0) = 0 \Rightarrow x = 0, y = 0$

$$\Rightarrow \tan^{-1}y = x + \frac{x^2}{2}$$

$$\Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$$

10. **(d)** None of these

Explanation: Given that $\vec{a} + \vec{b}$ is collinear with $\vec{c}$

$\therefore \vec{a} + \vec{b} = x\vec{c}$ ...(i)

where x is scalar and $x \neq 0$

$\vec{b} + \vec{c}$ is collinear with $\vec{a}$

$\vec{b} + \vec{c} = y\vec{a}$ ...(ii)

y is scalar and $y \neq 0$

Subtracting (ii) from (i) we get

$\vec{a} - \vec{c} = x\vec{c} - y\vec{a}$

$\vec{a} + y\vec{a} = x\vec{c} + \vec{c}$

$\vec{a}(1 + y) = (1 + x)\vec{c}$

As given

$\vec{a}, \vec{c}$ are not collinear. (no two vecotors are collinear)

$\therefore$ 1 + y = 0 and 1 + x = 0

y = -1 and x = -1

Putting value of x in equation (i)

$\vec{a} + \vec{b} = -\vec{c}$

$\vec{a} + \vec{b} + \vec{c} = 0$

11. **(d)** - log |cos x – sin x| + C

Explanation:

The integral is $\int \frac{(1+\tan x)}{(1-\tan x)} dx$

since we know that, $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

sin (a + b) = sin a cos b + cos a sin b

$\int$cot x = log (sin x) + c

Therefore,

$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\cos x+\sin x}{\cos x-\sin x} dx$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$\int \frac{-dt}{t} = -\log t + c$

$\Rightarrow$ -log |cos x - sin x| + c

12. **(b)** x (y + cos x) = sin x + c

Explanation: We have, $\frac{dy}{dx} + \frac{1}{x} y = \sin x$

Which is linear differential equation.

Here, P = $\frac{1}{x}$ and Q = sin x

$\therefore$ I.F. $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$\therefore$ The general solution is

$y \cdot x = \int x \cdot \sin x dx + C$

= - x cos x - $\int -\cos x dx$

= -x cos x + sin x

$\Rightarrow$ x(y + cos x) = sin x + C

13. **(d)** y = 2x - 4

Explanation: Let, $\frac{dy}{dx} = p$

$\therefore p^2 - xp + y = 0$

$y = xp - p^2$.... (i)

$\Rightarrow \frac{dy}{dx} = (x-2p)\frac{dy}{dx} + p$

$\Rightarrow p = (x-2p)\frac{dp}{dx} + p$

$\therefore \frac{dp}{dx} = 0$

$\Rightarrow$ P is constant

from Eqn. (i), $y = x \cdot c - c^2$

$\therefore$ $y = 2x - 4$ is the correct option

14. **(a)** $\frac{-e^{1/x}}{x^2}$

Explanation: Here $y = e^{\frac{1}{x}}$

Taking log both sides, we get

$\log_e y = \frac{1}{x}$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2}$ or $\frac{dy}{dx} = -\frac{1}{x^2} \times y$

Therefore, $\frac{dy}{dx} = -\frac{1}{x^2} \times e^{\frac{1}{x}}$

15. **(c)** Both (i) and (ii)

Explanation:

i. We have, $\frac{dy}{dx} = \text{f(x)} + \text{x} \Rightarrow \text{dy} = [\text{f(x)} + \text{x}]\text{dx}$

On integrating both sides, we get

$\int dy = \int[f(x) + x]\text{dx} \Rightarrow \text{y} = \int f(x)\,\text{dx} + \frac{x^2}{2} + \text{C}$

Let $\text{g(x)} = \int f(x)\text{dx} + \frac{x^2}{2}$

Thus, general solution is of the form y = g (x) + C

ii. Consider the given differential equation $\left(\frac{dy}{dx}\right)^2 = \text{f(x)}$

Clearly, the highest order derivative occurring in the differential equation is $\frac{dy}{dx}$ and its highest power is 2.

iii. Also, given equation is polynomial in the derivative. So the degree of a differential equation is 2.

16. **(a)** 8

Explanation: $(\text{adj}A) = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$

$= 8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= |A|\, I$

$|A| = 8.$

17. **(d)** $\frac{-\pi}{3}$

Explanation: Let the principle value be given by x

also, let $x = \tan^{-1}(-\sqrt{3})$

$\Rightarrow \tan x = -\sqrt{3}$

$\Rightarrow \tan x = -\tan\left(\frac{\pi}{3}\right)\left(\because \tan\left(\frac{\pi}{3}\right) = -\sqrt{3}\right)$

$\Rightarrow \tan x = \tan\left(-\frac{\pi}{3}\right)(\because -\tan(\theta) = \tan(-\theta))$

$\Rightarrow x = -\frac{\pi}{3}$

18. **(a)** perpendicular to z-axis

Explanation: We have,

$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

Also, the given line is parallel to the vector $\vec{b} = 3\hat{i} + \hat{j} + 0\hat{k}$

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

$3x + 4y + 0z = 0$

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1)

Hence, the given line is perpendicular to z-axis.

19. **(d)** A is false but R is true.

Explanation: Assertion: Here, $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 \sin\frac{1}{x}$

Putting $x = 0 - h$ as $x \rightarrow 0^-$, $h \rightarrow 0$

$\therefore \lim_{h\to 0} (0-h)^2 \sin\left(\frac{1}{0-h}\right) = \lim_{h\to 0} (-h^2 \sin\frac{1}{h})\ [\because \sin(-\theta) = -\sin\theta]$

$= -0 \times \sin(\infty)$

$= -0 \times$ (a finite value between -1 and 1)

$= 0$

$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 \sin\frac{1}{x}$

Putting $x = 0 + h$, as $x \rightarrow 0^+$, $h \rightarrow 0$

$\therefore \lim_{h\to 0} (0+h)^2 \sin\left(\frac{1}{0+h}\right) = \lim_{h\to 0} h^2 \sin\frac{1}{h}$

$= 0 \times \sin(\infty)$

$= 0 \times$ (a finite value between -1 and 1)

$= 0$

Also, f(0) = 0

$\therefore$ LHL = RHL = f(0).

Thus, f(x) is continuous at x = 0.

Reason Here, $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$

LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (\sin x - \cos x)$

Putting x = 0 - h as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{h\to 0} [\sin(0-h) - \cos(0-h)]$

$= \lim_{h\to 0} (-\sin h - \cos h)$

$= 0 - 1 = -1$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (\sin x - \cos x)$

Putting x = 0 + h as $x \to 0^+$ when $h \to 0$

$\therefore \lim_{h\to 0} [\sin(0+h) - \cos(0+h)]$

$= \lim_{h\to 0} (\sin h - \cos h)$

$= 0 - 1 = -1$

Also, f(0) = -1

$\therefore$ LHL = RHL = f(0).

Thus, f(x) is continuous at x = 0.

We know, when x < 0, f(x) = sin x - cos x is continuous and when x > 0, f(x) = sin x - cos x is also continuous.

Hence, f(x) is continuous for all values of x.

20. **(d)** A is false but R is true.

Explanation: Assertion: Shaded region shown as OABC is bounded and coordinates of its comer points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively.

Corner points	Corresponding value of Z = 13x -15y
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	-30 ← Minimum

Hence, the minimum value of objective function is at corner point (0, 2) which is - 30.

Hence, Assertion is not true.

Section B

21. Given that,

x = a sin t - b cot t; y = a cos t + b sin t

Differentiating both w.r.t. t

$$\Rightarrow \frac{dx}{dt} = \text{a cos t + b sin t}; \frac{dy}{dt} = \text{-a sin t + b cos t}$$

$$\Rightarrow \frac{dx}{dt} = y \text{ .....(i)}$$

$$\Rightarrow \frac{dy}{dt} = -x \text{ .....(ii)}$$

Dividing (ii) by (i)

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\frac{x}{y}$$

Differentiating both sides w.r.t. t

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2}\right\}$$

Putting values from (i) and (ii)

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y^2+x^2}{y^2}\right\} \text{ .....(iii)}$$

Dividing (iii) by (i)

$$\Rightarrow \frac{d^2y}{dx^2} = -\left\{\frac{y^2+x^2}{y^2 \times y}\right\} = -\left\{\frac{x^2+y^2}{y^3}\right\}$$

Hence proved

22. $\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\hat{k}$

$\vec{b}_2 = -\hat{i} + 8\hat{j} + 4\hat{k}$

$$\cos\theta = \left|\frac{\vec{b}_1 . \vec{b}_2}{\left|\vec{b}_1\right| \left|\vec{b}_2\right|}\right|$$

$$= \left|\frac{2(-1) + 5(8) + (-3)(4)}{\sqrt{38}\sqrt{81}}\right|$$

$$= \frac{26}{9\sqrt{38}}$$

$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

OR

Here,it is given that vector $\vec{r} = (4\hat{i} + 8\hat{j} + \hat{k})$

To find: angle between the vector and the x-axis

Formula used: If the vector $l\vec{i} + m\vec{j} + n\vec{k}$ and x- axis then the angle between the lines 'θ' is given by

$$\theta = \cos^{-1}\frac{l}{\sqrt{l^2+m^2+n^2}}$$

Here l = 4, m = 8, n = 1

$$\theta = \cos^{-1}\frac{4}{\sqrt{4^2+8^2+1^2}} = \cos^{-1}\frac{4}{\sqrt{16+64+1}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{81}} = \cos^{-1}\frac{4}{9}$$

$$\theta = \cos^{-1}\frac{4}{9}$$

thereforeThe angle between the vector and the x-axis is $\cos^{-1}\frac{4}{9}$

23. We have, $\frac{y}{1+y^2}dy = \frac{x}{1-x^2}dx$

Multiply 2 on both sides of equation,

$$\frac{2y}{1+y^2}dy = \frac{2x}{1-x^2}dx$$

Integrating on both the sides,

$$\int\frac{2y}{1+y^2}dy = \int\frac{2x}{1-x^2}dx$$

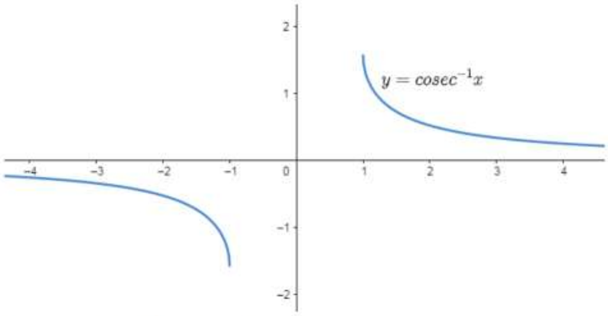
$\log(1 + y^2) = -\log(1- x^2) + \log c$

$\log(1 + y^2) = + \log(1- x^2) + \log c$

$=(1 + y^2) . (1 - x^2) = C.$

This is the required solution.

24. we know that Principal value branch of $\operatorname{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ and its graph is given below.



25. Consider the following events:

A = Selecting a fair-complexioned student.

B = Selecting a rich student.

C = Selecting a girl.

Clearly, $P(A) = \frac{20}{80} = \frac{1}{4}$, $P(B) = \frac{10}{80} = \frac{1}{8}$ and $P(C) = \frac{25}{80} = \frac{5}{16}$

Required probability is given by,

$P(A \cap B \cap C)$

$= P(A)\,P(B)\,P(C)$ [$\because$ A, B, C are independent events]

$= \frac{1}{4} \times \frac{1}{8} \times \frac{5}{16} = \frac{5}{512}$

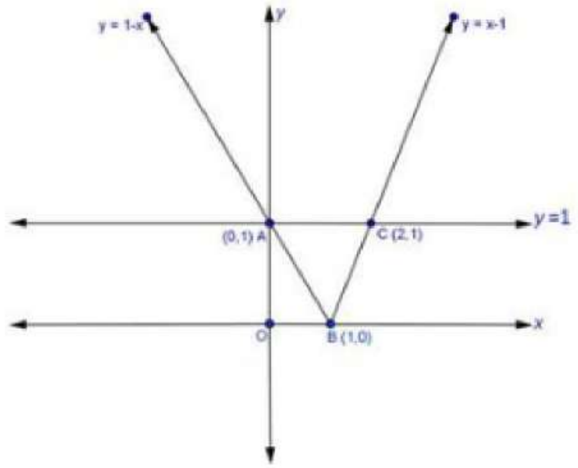
Section C

26. To find area bounded by y = 1 and

$y = |x - 1|$

$$y = \begin{cases} x - 1, & \text{if } x \geq 0 \quad \ldots(1) \\ 1 - x, & \text{if } x < 0 \quad \ldots(2) \end{cases}$$

A rough sketch of the curve is as under:-



Bounded reigon is the required region. So

Required area of bounded region=Area of Region ABCA

A = Region ABDA + Region BCDB

$= \int_0^1 \left(y_1 - y_2\right)dx + \int_1^2 \left(y_1 - y_3\right)dx$

$= \int_0^1 (1 - 1 + x)dx + \int_1^2 (1 - x + 1)dx$

$= \int_0^1 xdx + \int_1^2 (2 - x)dx$

$= \left(\frac{x^2}{2}\right)_0^1 + \left(2x - \frac{x^2}{2}\right)_1^2$

$$= \left(\frac{1}{2} - 0\right) + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right]$$

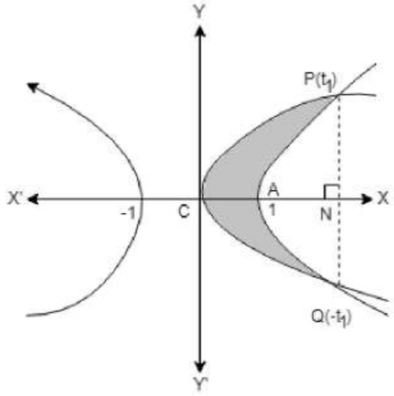
$$= \frac{1}{2} + \left(2 - 2 + \frac{1}{2}\right)$$

A = 1 sq. unit

OR

Let $P = \left[\frac{e^{t_1}+e^{-t_1}}{2}, \frac{e^{t_1}-e^{-t_1}}{2}\right]$ and $Q = \left[\frac{e^{-t}+e^{t_1}}{2}, \frac{e^{-t_1}-e^{t}}{2}\right]$

The required area = The area of the region bounded by the curves $x^2 - y^2 = 1$ and the lines joining the centre x = 0, y = 0 to the points t_1 and $-t_1$.



Required area= $= 2\left[area\ of\ \Delta PCN - \int_0^{\frac{e^{t_1}+e^{-t_1}}{2}} y dx\right]$

$$= 2\left[\frac{1}{2}\left(\frac{e^{t_1}+e^{-t_1}}{2}\right)\left(\frac{e^{t_1}-e^{-t_1}}{2}\right) - \int_0^{t_1} y \frac{dy}{dt}.\, dt\right]$$

$(\int \frac{dy}{dt} = y + c)$

$$= \frac{e^{2t_1}-e^{-2t_1}}{4} - \frac{1}{4} - \int_0^{t_1} (e^{2t} + e^{-2t} - 2)dt$$

$(\int e^t dt = e^t + c)$

$$= \frac{e^{2t_1}-e^{-2t_1}}{4} - \frac{1}{2}\left[\frac{e^{2t}}{2} - \frac{e^{-2t}}{2} - 2t\right]$$

$$= \frac{1}{4}(e^{2t_1} - e^{-2t_1} - e^{2t_1} + e^{-2t_1} + 4t_1)$$

$= t_1$ sq. units

The required area = The area of the region bounded by the curves $x^2 - y^2 = 1$ and the lines joining the centre x = 0, y = 0 to the points t_1 and $-t_1$. = t_1 sq. units

27. We have to maximize Z= 60 x + 15y First, we will convert the given inequations into equations, we obtain the following equations:

x + y = 50, 3x + y = 90, x = 0 and y = 0

Region represented by $x + y \leq 50$:

The line x + y = 50 meets the coordinate axes at A(50, 0) and B(0, 50) respectively. By joining these points we obtain the line 3x + 5 y = 15 Clearly (0, 0) satisfies the inequation $x + y \leq 50$. Therefore, the region containing the origin represents the solution set of the inequation $x + y \leq 50$

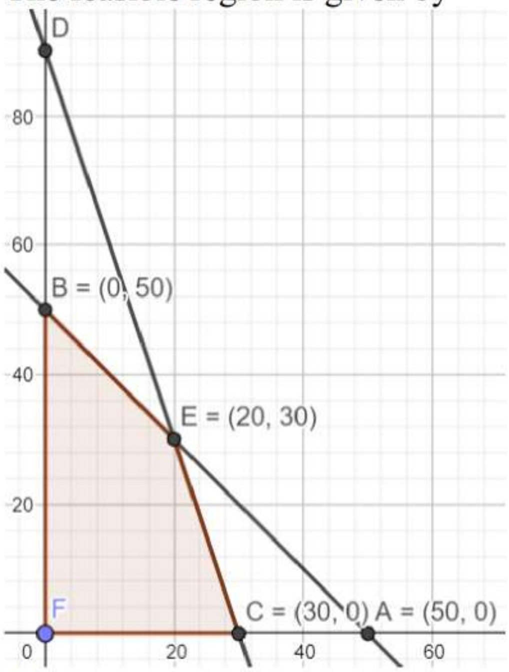
Region represented by $3x + y \leq 90$:

The line 3x + y = 90 meets the coordinate axes at C(30, 0) and D(0, 90) respectively. By joining these points we obtain the line 3x + y = 90 Clearly (0, 0) satisfies the inequation $3x + y \leq 90$. Therefore, the region containing the origin represents the solution set of the inequation $3x + y \leq 90$

Region represented by $x \geq 0$ and $y \geq 0$:

since, every point in the first quadrant satisfies these inequations. Therefore, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$.

The feasible region is given by



The corner points of the feasible region are O(0, 0), C(30, 0) E(20, 30) and B(0, 50)

The values of Z at these corner points are as follows given by

Corner point Z = 60 x + 15 y

O(0, 0) : $60 \times 0 + 15 \times 0 = 0$

C(30, 0) : $60 \times 30 + 15 \times 0 = 1800$

E(20, 30) : $60 \times 20 + 15 \times 30 = 1650$

B(0, 50) : $60 \times 0 + 15 \times 50 = 750$

Therefore, the maximum value of Z is 1800 at the point (30, 0) Hence, x = 30 and y = 0 is the optimal solution of the given LPP. Thus, the optimal value of Z is 1800.This is the required solution.

28. Given

$\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

Here,we have

$\vec{a}_1 = 3\hat{i} - 15\hat{j} + 9\hat{k}$

$\vec{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\vec{a_2} = -\hat{\imath} + \hat{\jmath} + 9\hat{k}$

$\vec{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

Thus,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$

$\therefore \vec{b_1} \times \vec{b_2} = 17i + 16\hat{j} + 16\hat{k}$

$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{17^2 + 16^2 + 17^2}$

$= \sqrt{289 + 256 + 289}$

$= \sqrt{834}$

$\vec{a_2} - \vec{a_1} = (-1 - 3)\hat{i} + (1 + 15)\hat{j} + (9 - 9)\hat{k}$

$\therefore \vec{a_2} - \vec{a_1} = -4\hat{i} + 16\hat{j} + 0\hat{k}$

= -68 + 256 + 0

= 188

Thus, the shortest distance between the given lines is

$$d = \left| \frac{\left(\vec{b}_1 \times \vec{b_2}\right)}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{188}{\sqrt{834}} \right|$$

$$\therefore d = \frac{188}{\sqrt{834}} \text{ units}$$

As $d \neq 0$

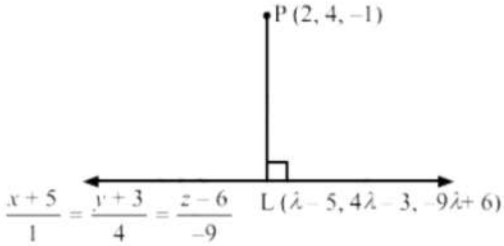
Thus, the given lines do not intersect.

OR

Suppose L be the foot of the perpendicular drawn from the point P(2,4,-1) to the given line.

The coordinates of a general point on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ are given by

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$$
$\Rightarrow x=\lambda-5$
$y=4\lambda-3$
$z=-9\lambda+6$
Suppose the coordinates of L be $(\lambda-5, 4\lambda-3, -9\lambda+6)$



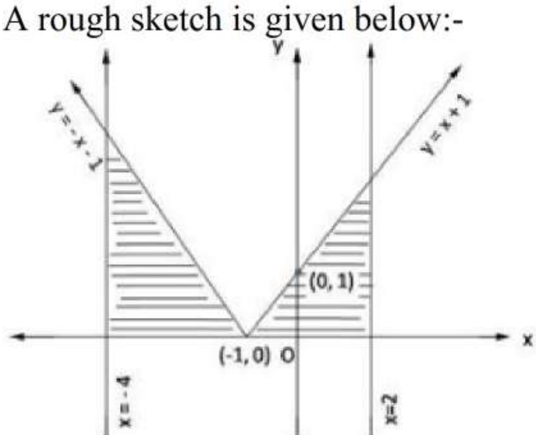
The direction ratios of PL are proportional to
$\lambda-5-2, 4\lambda-3-4, -9\lambda+6+1$, i.e. $\lambda-7, 4\lambda-7, -9\lambda+7$
Now the ratios of the given line are proportional to 1,4,-9, but PL is perpendicular to the given line.
$\therefore 1(\lambda-7)+4(4\lambda-7)-9(-9\lambda+7)=0$
$\Rightarrow \lambda=1$
Put
$\Rightarrow \lambda=1$ in $(\lambda-5, 4\lambda-3, -9\lambda+6)$ we get the coordinates of L as (-4,1,-3)
Equation of the line PL is
$$\frac{x-2}{-4-2}=\frac{y-4}{1-4}=\frac{z+1}{-3+1}$$
or $\frac{x-2}{-6}=\frac{y-4}{-3}=\frac{z+1}{-2}$

29. We have,

$$y=|x+1|=\begin{cases}x+1, & \text{if } x+1\geq 0\\ -(x+1), & \text{if } x+1<0\end{cases}$$

$$y=\begin{cases}(x+1), & \text{if } x\geq -1\\ -x-1, & \text{if } x<-1\end{cases}$$

$\Rightarrow$ y = x + 1 ...(i)
and y = - x - 1 ...(ii)
Equation (i) represent a line which meets axes at (0, 1) and (-1, 0). Equation (ii) represent a line passing through (0, -1) and (-1, 0)
A rough sketch is given below:-



$$\int_{-4}^{2} |x+1|\,dx = \int_{-4}^{-1} -(x+1)dx + \int_{-1}^{2} (x+1)dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-4}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{16}{2}-4\right)\right]+\left[\left(\frac{4}{2}+2\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}-4\right)\right]+\left[4+\frac{1}{2}\right]$$

$$= \frac{9}{2}+\frac{9}{2}$$

$$= \frac{18}{2}$$

Required area = 9 sq. unit

This integral represents the area under the shaded region.

30. Let $I = \int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$ ...(i)

Let $x^2 + \sin 2x + 2x = t$ then,

$d(x^2 + \sin 2x + 2x) = dt$

$$\Rightarrow \quad dx = \frac{dt}{2(\cos 2x + x + 1)}$$

Putting $x^2 + \sin 2x + 2x = t$ and $dx = \frac{dt}{2(\cos 2x + x + 1)}$ in equation (i), we get

$$I = \int \frac{\cos 2x + x + 1}{t} \times \frac{dt}{2(\cos 2x + x + 1)}$$

$$= \frac{1}{2}\int \frac{dt}{t}$$

$$= \frac{1}{2}\log|t| + c$$

$$= \frac{1}{2}\log|x^2 + \sin 2x + 2x| + C$$

$\therefore\ I = \frac{1}{2}\log|x^2 + \sin 2x + 2x| + C$, which is the value of the given integral.

OR

Let the given integral be ,

$$I = \int \frac{x^3 \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

Let $\sin^{-1} x^2 = t$

$$\frac{1}{\sqrt{1-x^4}} (2x)\, dx = dt$$

$$I = \int \frac{x^2 \sin^{-1} x^2}{\sqrt{1-x^2}} x\, dx$$

$$= \int (\sin t) t \frac{dt}{2}$$

$$= \frac{1}{2} \int t \sin t\, dt$$

$$= \frac{1}{2}[t \int \sin t\, dt - \int (1 \int \sin t\, dt)\, dt]$$

$$= \frac{1}{2}[t (-\cos t) - \int (-\cos t)\, dt]$$

$$= \frac{1}{2}[-t \cos t + \sin t] + c$$

$$I = \frac{1}{2}[x^2 - \sqrt{1-x^4} \sin^{-1} x^2] + c$$

31. $x = ae^t (\sin t + \cos t)$ and $y = ae^t (\sin t - \cos t)$ (Given)

$$\therefore \frac{dx}{dt} = a[e^t(\cos t - \sin t) + e^t(\sin t + \cos t)]$$

$= -y + x$

and $$\frac{dy}{dt} = a[e^t (\sin t - \cos t) + e^t(\sin t + \cos t)]$$

$= y + x$

Therefore

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

or $$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Section D

32. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

$$\therefore |A| = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

$$= \left(-\cos^2\alpha - \sin^2\alpha\right) - 0 + 0 = -\left(\cos^2\alpha + \sin^2\alpha\right) = -1 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = +\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} = +\left(-\cos^2\alpha - \sin^2\alpha\right)$$

$$= -\left(\cos^2\alpha + \sin^2\alpha\right) = -1$$

$$A_{12} = -\begin{vmatrix} 0 & \sin\alpha \\ 0 & -\cos\alpha \end{vmatrix} = -(0-0) = 0,\; A_{13} = +\begin{vmatrix} 0 & \cos\alpha \\ 0 & \sin\alpha \end{vmatrix} = +(0-0) = 0$$

$$A_{21} = -\begin{vmatrix} 0 & 0 \\ \sin\alpha & -\cos\alpha \end{vmatrix} = -(0-0) = 0,$$

$$A_{22} = +\begin{vmatrix} 1 & 0 \\ 0 & -\cos\alpha \end{vmatrix} = +(-\cos\alpha - 0) = -\cos\alpha$$

$$A_{23} = -\begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -(\sin\alpha - 0) = \sin\alpha,\; A_{31} = +\begin{vmatrix} 0 & 0 \\ \cos\alpha & \sin\alpha \end{vmatrix} = (0-0) = 0$$

$$A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -(\sin\alpha - 0) = -\sin\alpha,$$

$$A_{33} = +\begin{vmatrix} 1 & 0 \\ 0 & \cos\alpha \end{vmatrix} = +(\cos\alpha - 0) = \cos\alpha$$

$$\therefore adj.A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj.A = -\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & -\cos\alpha \end{bmatrix}$$

OR

For the given system of equations, we have

$$D = \begin{vmatrix} 2 & a & 6 \\ 1 & 2 & b \\ 1 & 1 & 3 \end{vmatrix}$$

$\Rightarrow$ D = 2(6 - b) -a(3 - b) + 6(1 - 2)

$\Rightarrow$ D = 12 - 2b - 3a + ab - 6 = 6 - 3a - 2b + ab = (b - 3)(a - 2)

$$D_1 = \begin{vmatrix} 8 & a & 6 \\ 5 & 2 & b \\ 4 & 1 & 3 \end{vmatrix}$$

$\Rightarrow$ D_1 = 8(6 - b) - a(15 - 4b) + 6(5 - 8)

$\Rightarrow$ D_1 = 48 − 8b − 15a + 4ab − 18 = 30 − 15a − 8b + 4ab = (a − 2)(4b − 15)

$$D_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & b \\ 1 & 4 & 3 \end{vmatrix}$$

$\Rightarrow$ D_2 = 2(15 − 4b) − 8(3 − b) + 6(4 − 5) = 30 − 8b − 24 + 8b − 6 = 0

and, $D_3 = \begin{vmatrix} 2 & a & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix}$ = 2(8 − 5) − a(4 − 5) + 8(1 − 2) = 6 + a − 8 = a − 2

i. For unique solution, we must have
D $\neq$ 0 $\Rightarrow$ (a - 2) (b - 3) $\neq$ 0 $\Rightarrow$ a $\neq$ 2, b $\neq$ 3

ii. For infinitely many solutions, we must have
D = D_1 = D_2 = D_3 = 0
$\Rightarrow$ (a - 2)(b - 3) = 0, (a - 2)(4b - 15) = 0 and a - 2 = 0
$\Rightarrow$ a = 2
Putting a = 2 in the given system of equations, we obtain
2x + 2y + 6z = 8
x + 2y + bz = 5
x + y + 3z = 4
This system is equivalent to the system
x + y + 3z = 4
x + 2y + bz = 5
Putting z = k, we get
x + y = 4 - 3k
x + 2y = 5 - bk
Solving these two equations, we get
x = 3 - 6k + bk, y = 1 - bk + 3k
Thus, the given system has infinitely many solutions given by
x = 3 - 6k + bk, y = 1 - bk + 3k, z = k, where k $\in$ R.
Hence, the system has infinitely many solutions for a = 2

iii. For no solution, we must have

D = 0 and at least one of D_1, D_2 and D_3 is non-zero.

Clearly, for b = 3, we have

D = 0 and $D_3 \neq 0$.

Hence, the system has no solution for b = 3.

33. We have,

$$\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

$$\vec{a} \times \vec{b} = \left(\frac{1}{7}\right)\left(\frac{1}{7}\right)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \frac{1}{49}(42\hat{i} + 14\hat{j} - 21\hat{k})$$

$$= \frac{1}{49}[7(6\hat{i} + 2\hat{j} - 3\hat{k})]$$

$$= \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

$$= \vec{c}$$

$$\vec{b} \times \vec{c} = \left(\frac{1}{7}\right)\left(\frac{1}{7}\right)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 6 & 2 & -3 \end{vmatrix}$$

$$= \frac{1}{49}(14\hat{i} + 21\hat{j} + 42\hat{k})$$

$$= \frac{1}{49}[7(2\hat{i} + 3\hat{j} + 6\hat{k})]$$

$$= \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$= \vec{a}$$

$$\vec{c} \times \vec{a} = \left(\frac{1}{7}\right)\left(\frac{1}{7}\right)\begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 2 & -3 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \frac{1}{49}(21\hat{i} - 42\hat{j} + 14\hat{k})$$

$$= \frac{1}{49}[7(3\hat{i} - 6\hat{j} + 2\hat{k})]$$

$$= \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$

$$= \vec{b}$$

$$|\vec{a}| = \frac{1}{7}\sqrt{4 + 9 + 36}$$

$$= \frac{7}{7}$$

$$= 1$$

$$|\vec{b}| = \frac{1}{7}\sqrt{9 + 36 + 4}$$

$$= \frac{7}{7}$$

$$= 1$$

$$|\vec{c}| = \frac{1}{7}\sqrt{36 + 4 + 9}$$

$$= \frac{7}{7}$$

$$= 1$$

Therefore, $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a right handed orthogonal system of units vectors.

34. Let $I = \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right)e^{2x}dx \ldots$(i)

Putting 2x = t

$$\Rightarrow 2 = \frac{dt}{dx}$$

$$\Rightarrow 2dx = dt$$

$$\Rightarrow dx = \frac{dt}{2}$$

Limits of integration when $x = 1, t = 2 \times 1 = 2$ and when $x = 2, t = 2 \times 2 = 4$

$\therefore$ From eq. (i),

$$I = \int_2^4 \left(\frac{1}{\frac{t}{2}} - \frac{1}{2\left(\frac{t}{2}\right)^2}\right)e^t \frac{dt}{2}$$

$$= \int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right)e^t \frac{dt}{2}$$

$$= \int_2^4 \frac{1}{2}.2\left(\frac{1}{t} - \frac{1}{t^2}\right)e^t dt$$

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right)e^t dt$$

$$= \int_2^4 \left\{f(t) + f'(t)\right\}e^t dt$$

$$= \left\{e^t f(t)\right\}_2^4$$

$$= \left(\frac{e^t}{t}\right)_2^4$$

$$= \frac{e^4}{4} - \frac{e^2}{2}$$

$$= \frac{e^4 - 2e^2}{4}$$

35. R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivty

For any a ∈ A,|a−a|=0 Which is divisible by 2.

∴ (a, a) ∈ r for all a ∈ A

So ,R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all a, b ∈ R

|a−b| is divisible by 2

|b−a| is divisible by 2

$(a, b) \in r \Rightarrow (b, a) \in R$

So, R is symmetirc .

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

|a−b| is divisible by 2

|b−c| is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

∴ (a, c) ∈ R

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence , R is an equivalence relation

OR

We observe the following properties of relation R.

Reflexivity: For any a $\in$ N

a - a = 0 = 0 × n

⇒ a - a is divisible by n

⇒ (a, a) $\in$ R

Thus, (a, a) $\in$ for all a $\in$ Z. So, R is reflexive on Z

Symmetry: Let (a, b) $\in$ R. Then,

(a, b) $\in$ R

⇒ (a - b) is divisible by n

⇒ (a - b) = np for some p $\in$ Z

⇒ b - a = n (-p)

⇒ b - a is divisible by n [$\because p \in Z \Rightarrow -p \in Z$]

⇒ (b, a) $\in$ R

Thus, (a, b) $\in$ R ⇒ (b, a) $\in$ R for all a, b $\in$ Z.

So, R is symmetric on Z.

Transitivity: Let a, b, c $\in$ Z such that (a, b) $\in$ R and (b, c) $\in$ R. Then,

(a, b) $\in$ R

⇒ (a - b) is divisible by n

⇒ a - b = np for some p $\in$ Z

and, (b, c) $\in$ R

⇒ (b - c) is divisible by n

⇒ b - c = nq for some q $\in$ Z

∴ (a, b) $\in$ R and (b, c) $\in$ R

⇒ a - b = np and b - c = nq

⇒ (a - b) + (b - c) = np + nq

⇒ a - c = n (p + q)

⇒ a - c is divisible by n [$\because p, q \in Z \Rightarrow p + q \in Z$]

⇒ (a, c) $\in$ R

Thus, (a, b) $\in$ R and (b, c) $\in$ R ⇒ (a, c) $\in$ R for all a, b, c $\in$ Z.

Section E

36. **Read the text carefully and answer the questions:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm.

The volume V(x) of the box is given by $V(x) = x(18 - x)^2$

(ii) $V(x) = x(18 - 2x)^2$

$$\frac{dV(x)}{dx} = (18 - 2x)^2 - 4x(18 - 2x)$$

For maxima or minima = $\frac{dV(x)}{dx} = 0$

$\Rightarrow (18 - 2x)[18 - 2x - 4x] = 0$

$\Rightarrow x = 9$ or $x = 3$

$\Rightarrow x =$ not possible

$\Rightarrow x = 3$ cm

The side of the square to be cut off so that the volume of the box is maximum is x = 3 cm

(iii) $$\frac{dV(x)}{dx} = (18 - 2x)(18 - 6x)$$

$$\frac{d^2V(x)}{dx^2} = (18 - 6x)(-2) + (18 - 2x)(-6)$$

$$\Rightarrow \frac{d^2V(x)}{dx^2} = -12[3 - x + 9 - x] = -24(6 - x)$$

$$\Rightarrow \left.\frac{d^2V(x)}{dx^2}\right]_{x=3} = -72 < 0$$

$\Rightarrow$ volume is maximum at x = 3

OR

$V(x) = x(18 - 2x)^2$

When x = 3

$V(3) = 3(18 - 2 \times 3)^2$

$\Rightarrow$ Volume $= 3 \times 12 \times 12 = 432\ cm^3$

37. **Read the text carefully and answer the questions:**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

$E_1 = PQ$, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$

$$P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) = 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1$$

$$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)$$

$= 0.14 + 0.24 + = 0.38$

(ii)

By Bayes' Theorem, $P\left(\frac{E_3}{E}\right) =$

$$\frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}$$

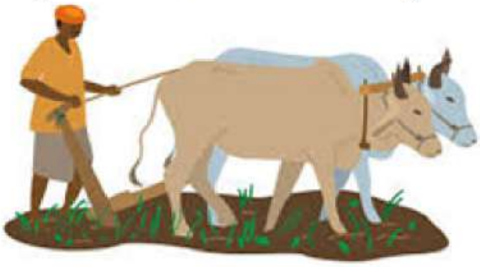
$$= \frac{0.14}{0.38} = \frac{7}{19}$$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated

as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$

38. **Read the text carefully and answer the questions:**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{pmatrix} \textit{Urad} & \textit{Masoor} & \textit{Mung} \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \textit{Ankit} \\ \textit{Girish} \end{matrix}$$

October sales (in ₹):

$$A = \begin{pmatrix} \textit{Urad} & \textit{Masoor} & \textit{Mung} \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \textit{Ankit} \\ \textit{Girish} \end{matrix}$$

(i)

$$A + B = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$$

The combined sales of Masoor in September and October, for farmer Girish ₹40000.

(ii)

$$A + B = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$$

The combined sales of Urad in September and October, for farmer Ankit is ₹15000.

(iii)

$$A - B = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} - \begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$$

$$= \begin{bmatrix} 10,000 - 5000 & 20,000 - 10,000 & 30,000 - 6000 \\ 50,000 - 20,000 & 30,000 - 10,000 & 10,000 - 10,000 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 5000 & 10,000 & 24,000 \\ 30,000 & 20,000 & 0 \end{bmatrix} \begin{matrix} \text{Ankit} \\ \text{Girish} \end{matrix}$$

OR

Profit = 2% $\times$ sales on october

$$= \frac{2}{100} \times B$$

$$= 0.02 \times \begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$$

$$= \begin{bmatrix} 0.02 \times 5000 & 0.02 \times 10,000 & 0.02 \times 6000 \\ 0.02 \times 20,000 & 0.02 \times 10,000 & 0.02 \times 10,000 \end{bmatrix}$$

$$= \begin{bmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{bmatrix} \begin{matrix} \text{Ankit} \\ \text{Girish} \end{matrix}$$